▸ PETER M. GERDES, *Sets with a non-uniform self-modulus.*

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For $f, g \in \omega^\omega$ denote g pointwise dominates f by $g \succ f$. Following Slaman and Groszek[1] say f is a modulus (of computation) of X if every $g \succ f$ can compute X and that f is a self-modulus if f is a modulus for some X with $X \equiv_T f$. A modulus f of X is uniform if there is a common reduction that allows every $g \succ f$ to compute X. If X has a modulus than X is recursively encodable so by a result of Solovay[2] X must be Δ^1_1 and by [1] X must have a Δ^1_1 uniform modulus. While one can straightforwardly construct examples of uniform self-moduli and it is only slightly more difficult to find sets that lack a self-modulus it isn't obvious that a non-uniform self-modulus even exists. We present a construction, borrowing heavily from the machinery of forcing, of just such a function. Generalizing further for every n we construct an f such that $g \succ f \Longrightarrow g \geq_T f$ and if $h \leq_T f^{(n)}$ then h is not a uniform modulus for f. If time permits we will also discuss the general case where n is replaced with an arbitrary recursive ordinal.

[1] Marcia J. Groszek and Theodore A. Slaman, *Unpublished*

[2] Robert M. Solovay, *Hyperarithmetically encodable sets*, ***Transacttions of the American Mathematical Society***, vol. 239 (1978) pp. 99–122.